

Practice Integrals

If you would like answers, use Wolfram alpha (just type in, e.g., “Integrate $x^2 + 1$ ” and it should give you $\frac{x^3}{3} + x + C$). If you get stuck on how an integral works, there are some hints on the back.

1.

$$\int x e^x dx \quad \int \ln(t) dt \quad \int (r+2)^{5/2} dr \quad \int e^x \cos(x) dx \quad \int \cos(x) \sin(2x) dx$$

2.

$$\int (p+1)^3 (p+2)^2 dp \quad \int (\sin(y)+1)^2 dy \quad \int t^2 e^{-t} dt \quad \int \frac{1}{x^3} dx$$

3.

$$\int \frac{1}{x \ln(x)} \quad \int \frac{x}{(x^2-1)^{3/2}} \quad \int e^{\cos(5y)} \sin(5y) dy \quad \int \theta \sin(\pi + \theta^2) d\theta$$

4.

$$\int (1+s) \cos(s) ds \quad \int \cos(s^2) s ds \quad \int x \ln(x) dx \quad \int x^3 e^{x^2} dx$$

5.

$$\int 7x e^{-2x^2} dx \quad \int x^2 \sin\left(\frac{x}{2}\right) dx \quad \int \frac{5y}{\left(\frac{y}{4}\right)^2 - 1} dy$$

6.

$$\int e^{-x} \cos(5x) dx \quad \int \left(x^3 + 5x^2 - x - \frac{1}{2}\right) dx \quad \int \frac{x^6 - x^4 - x^2}{x^2} dx \quad \int \frac{\ln(y)}{y} dy \quad \int \frac{\ln(x)(x-1)}{x} dx$$

7.

$$\int (5x)^3 e^{5x} dx \quad \int (u^2+1) du \quad \int \frac{1}{u^2+1} du \quad \int \frac{1}{\sqrt{1-x^2}} dx \quad \int \frac{\frac{1}{2}x}{(x^2+1)^{3/2}}$$

8.

$$\int (1+t^2)^{-3/2} dt \quad \int \frac{x+1}{x} dx \quad \int \frac{\sin(\theta)}{\cos^2(\theta)+1} d\theta \quad \int \tan(x) dx$$

9.

$$\int_0^1 x^n dx \quad \int_0^\infty e^{-x} dx \quad \int_{-\infty}^\infty r e^{-r^2} dr \quad \int_{-\pi/4}^{\pi/4} \tan(x) dx$$

10.

$$\int_0^1 \sqrt{1-x^2} dx \quad \int_{-\infty}^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad \int_0^1 \sqrt{x} dx \quad \int_0^5 3e^{-\frac{x}{2}} x dx \quad \int_0^x \cos(t) dt \quad \int_0^1 dx$$

11.

$$\int_1^x \ln(t) dt \quad \int_2^x e^t dt \quad \int_0^x \frac{1}{1+s^2} ds \quad \int_0^x e^{-y} dy.$$

Selected Hints/Answers

1. $\int xe^x$ is integration by parts. $\int \ln(t)dt$ is also integration by parts:

$$\int \ln(t) = \int \ln(t) \left(\frac{d}{dt} t \right) dt = t \ln(t) - \int \left(\frac{d}{dt} \ln(t) \right) t dt.$$

$\int e^x \cos(x)dx$ is integration by parts, twice:

$$\begin{aligned} \int e^x \cos(x)dx &= \int \left(\frac{d}{dx} e^x \right) \cos(x)dx = e^x \cos(x) - \int e^x \left(\frac{d}{dx} \cos(x) \right) dx = e^x \cos(x) + \int e^x \sin(x) \\ &= e^x \cos(x) + \int \left(\frac{d}{dx} e^x \right) \sin(x)dx \\ &= e^x \cos(x) + \left[e^x \sin(x) - \int e^x \left(\frac{d}{dx} \sin(x) \right) \right] = e^x \cos(x) + \left[e^x \sin(x) - \int e^x \cos(x) \right] \end{aligned}$$

so

$$\int e^x \cos(x)dx = e^x \cos(x) + e^x \sin(x) - \int e^x \cos(x)dx$$

so

$$2 \int e^x \cos(x)dx = e^x \cos(x) + e^x \sin(x)$$

so

$$\int e^x \cos(x)dx = \frac{1}{2}(e^x \cos(x) + e^x \sin(x)) + C.$$

For $\int \cos(x) \sin(2x)dx$, note that $\sin(2x) = 2 \sin(x) \cos(x)$ (double angle formula for sine).

2. To integrate $\sin^2(y)$, use the half angle formula for sine. $\int t^2 e^{-t} dt$ is integration by parts.
3. The first is by parts, the next are all u -substitutions.
4. For the last one, do it by parts but split it like $x^3 e^{x^2} = x^2(xe^{x^2})$.
5. Second one is integration by parts. The others can be done with u -substitutions.
8. I think the first one is hard. I did it by parts after adding and subtracting something to the numerator. For $\int \frac{\sin(\theta)}{\cos^2(\theta)+1} d\theta$, use $u = \cos(\theta)$. For the last one, note that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ and do a u -substitution.